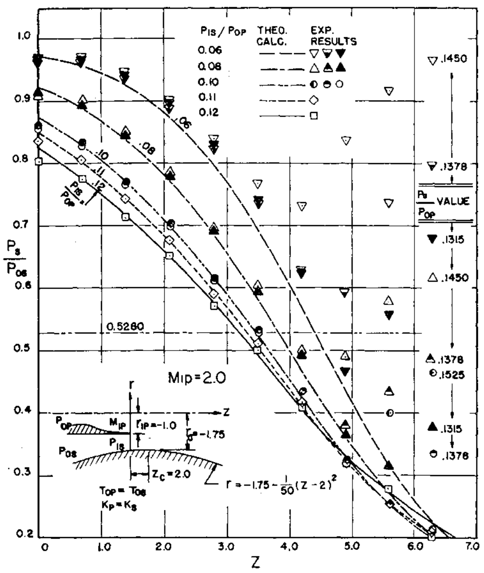

Fig. 5 Pressure distributions on the shroud wall of an ejector system.

out beyond the section where the secondary flow is choked by imposing the secondary stream to be supersonic. The crossover of various constant P_{1s}/P_{op} value curves in this region is reasonable, judging from the fact that an insufficient amount of the secondary flow rate (lower P_{1s}/P_{op} values) at the inlet will result in undesirable overexpansion at the downstream of the ejector-nozzle propulsive systems. Again, the experimental results show very good agreement with the theoretical calculations. Also reported in the same figure is a series of experimental results corresponding to higher ambient pressure ratios P_a/P_{op}§, so that they do not correspond to the supersonic regime. Nevertheless, these results indicate that the system behaves in a manner similar to a convergent-divergent nozzle operating at higher back pressures. It is also interesting to note that, at the various levels of the ambient pressure ratios (corresponding to different high primary stagnation pressures), the results are not much different. One may conclude that, at high Reynolds numbers, the influence of the Reynolds number on the characteristics of the system is indeed negligible.

In the interest of the current development of fluid jet control techniques and devices, this concept of interaction between the primary and secondary streams has also been employed to predict the characteristics of the two-dimensional ejector systems. It is worthwhile to mention that the boundary layer on the side walls of a two-dimensional ejector system may present a predominate modification to the performance of such an ejector system.

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§ A detailed study of the effect of this parameter would have to consider the over-all experimental ejector system.

Temperature Effect of Gaseous Hydrogen on Cooling Effectiveness

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Nomenclature

a = coolant flow area
c_{pc} = coolant specific heat
d_h = hydraulic diameter of coolant circuit
h_c = coolant-side heat-transfer coefficient
h_g = hot gas-side heat-transfer coefficient
k_c = thermal conductivity of coolant
k_w = thermal conductivity of wall
q/A = heat flux
t = wall thickness
T_c = coolant temperature
T_{wc} = coolant-side wall temperature
T_{wh} = hot gas-side wall temperature
T_{adw_g} = hot gas adiabatic wall temperature
w_c = coolant flow rate
θ = defined by Eq. (6)
μ_c = viscosity of coolant
ϕ = defined by Eq. (7)

Introduction

MANY ramjet and rocket engine designs use regenerative cooling to maintain the thrust chamber wall at temperatures consistent with good structural integrity. The fuel is generally used as the coolant and is first passed through the coolant circuit, then injected into the combustor as a gas. Hydrogen offers high performance and exceptional cooling qualities; however, any regenerative cooling circuit is limited to the amount of heat it can remove from the engine wall. One method of extending this limit is to design the regenerative cooling circuit so that an optimum coolant temperature occurs at the most critical point in the system. The results of this analysis establish the criteria to determine this optimum coolant temperature.

Analysis

1. General

Forced convection cooling is assumed. The coolant-side heat-transfer coefficient is given as[1]

$$h_c = 0.025 \frac{k_c}{d_h} \left(\frac{w_c d_h}{a\mu_c}\right)^{0.8} \left(\frac{c_{pc}\mu_c}{k_c}\right)^{0.4} \left(\frac{T_c}{T_{wc}}\right)^{0.55} \tag{1}$$

Properties of the coolant are based on bulk coolant temperature. The heat flux to the wall is[2]

$$q/A = h_c\,(T_{wc} - T_c) \tag{2}$$

$$q/A = h_g\,(T_{adw_g} - T_{wh}) \tag{3}$$

$$q/A = (k_w/t)(T_{wh} - T_{wc}) \tag{4}$$

where h_g is calculated using the analysis suggested by Ref. 3. The following assumptions are made: 1) all properties of the hot gas (combustion products) are known, and 2) $T_{wh_{max}}$ has been established consistent with the wall material chosen. Assumptions 1 and 2 allow the heat flux to the wall to be calculated using Eq. (3). Substituting Eq. (1) into Eq. (2) and solving for w_c yields

$$w_c = [\theta\phi/(T_{wc} - T_c)]^{1.25}\,(T_c)^{-0.6875} \tag{5}$$

where

$$\theta = \frac{(q/A)\;(d_h)^{0.2}\;(a)^{0.8}\;(T_{wc})^{0.55}}{0.025} \tag{6}$$

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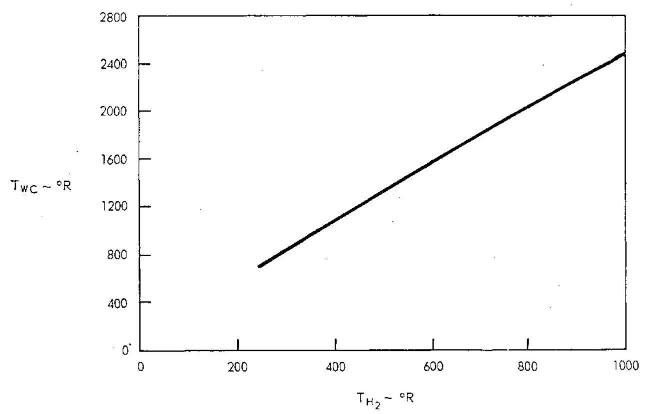

Fig. 1 Gaseous hydrogen coolant temperature to minimize flow for given T_{wc}.

$$\phi = (k_c)^{-0.6} (\mu_c/c_{pc})^{0.4} \quad (7)$$

For a given engine condition, with the dimensions of a coolant circuit fixed, θ = const. Equation (5) is differentiated to give

$$\frac{dw_c}{dT_c} = (\theta)^{1.25}(T_c)^{-0.6875}(T_{wc} - T_c)^{-1.25}(\phi)^{1.25} \times \left\{1.25\left[\frac{d\phi/dT_c}{\phi} + \frac{1}{T_{wc} - T_c}\right] - \frac{0.6875}{T_c}\right\} \quad (8)$$

$(dw_c/dT_c) = 0$ establishes the criterion for a minimum.

2. Analysis with gaseous Hydrogen

In order to solve Eq. (8), ϕ must be defined. Gaseous hydrogen at high pressure (500 psia) is assumed as the coolant. At high pressure c_{pc}, k_c, and μ_c are essentially independent of pressure. An approximate linear curve fit yields

$$\phi\,(T_{H_2}) \cong 0.521 - 7.73 \times 10^{-5}(T_{H_2}) \quad (9)$$

$$d\phi/dT_{H_2} \cong -7.73 \times 10^{-5} \quad (10)$$

Only one solution which is physically valid may be obtained from Eq. (8) when $(dw_c/dT_c) = 0$. The criterion for this solution is obtained from the term

$$1.25\left[\frac{d\phi/dT_{H_2}}{\phi} + \frac{1}{T_{wc} - T_{H_2}}\right] - \frac{0.6875}{T_{H_2}} = 0 \quad (11)$$

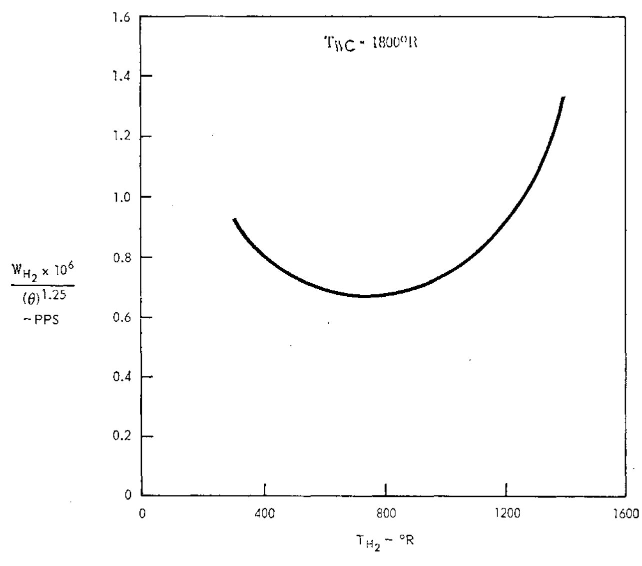

Fig. 2 Regenerative cooling flow variation with coolant temperature, T_{wc} = 1800°R.

Equation (11) is a quadratic in T_{H_2}, which yields the following solution

$$T_{H_2} = 1.90 \times 10^4 - 0.820(T_{wc}) - [3.61 \times 10^8 + 0.672 \times (T_{wc})^2 - 5.82 \times 10^4(T_{wc})]^{1/2} \quad (12)$$

Figure 1 is a plot of Eq. (12). Current hypersonic ramjet designs, utilizing regenerative cooling, employ T_{wc} values to 3000°R depending upon the application and the resulting material selection. For illustrative purposes a T_{wc} = 1800°R has been chosen. The variation of the coolant flow requirements with coolant (hydrogen) temperature at this condition is shown in Fig. 2.

Conclusions

A significant variation in hydrogen coolant flow requirements as a function of coolant temperature can occur in regeneratively cooled engines. Clever design of the coolant flow circuit can reduce the cooling flow rate more than 60% if the cryogenic hydrogen carried on the vehicle is properly elevated in temperature prior to cooling the throat of the nozzle.

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Further Similarity Solutions of Two-Dimensional Wakes and Jets

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SOLUTIONS of the Falkner-Skan equation

$$f''' + ff'' + \beta[1 - f'^2] = 0 \quad (1a)$$

subject to the boundary conditions

$$f(0) = f''(0) = 0 \qquad f'(\infty) = 1.0 \quad (1b)$$

describe the two-dimensional, isoenergetic, symmetric viscous free mixing with streamwise pressure gradient under conditions of similarity, wherein

$$u = u_e(s)f'(\eta)$$

$$\eta = \frac{u_e}{(2s)^{1/2}}\int_0^y \rho dy \qquad s = \int_0^x \rho_e\mu_e u_e dx$$

$$\beta = \frac{2s}{u_e}\frac{du_e}{ds}\frac{H_e}{h_e} \qquad \rho\mu = \rho_e\mu_e$$

Primes denote total differentiation with respect to η and sub-

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